Stability and transient-behavioural assessment of power-electronics based dc-distribution systems.


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Summary

Power electronic systems are becoming increasingly important in marine engineering applications. Power electronics is stimulating new concepts and developments in electrical machines and distribution systems.

Concomitant with such developments is usually a need for new methods for analysis and design activities. An increasing serious contender for a power distribution system is a dc one. Such systems are well-known in satellite engineering. Inevitably they will become more important as fuel-cells become common in marine use. One problem with dc-based systems is that they are susceptible to instability, particularly when a constant-power regime is operating. Work on examining this problem is described by Sudhoff et al. [11] using a method based on the Nyquist stability criterion. The purpose of this contribution is to demonstrate that an approach to this problem, based on the root-locus technique, is well worth considering. Not only does this technique produce indications of when to expect instability, it also indicates the type of behaviour to be expected at any operating point.

In this way it provides a simple means of assessing transient behaviour as the values of certain key components are altered. The efficacy of the technique is demonstrated by using a problem originally analysed in [11].
1. Introduction

Developments in power-electronic devices in the past couple of decades have created a renaissance of interest in researching, designing and testing novel electrical machines and power systems. Much of the new thinking, generated by these activities, is of considerable current and potential interest to the marine engineering community. For example, the electrical machines sector has seen interest in the advanced induction-motor, the transverse-flux motor and the switched-reluctance motor [1,2,3]. In the power distribution field the ability of taking electric power generated by, say, the prime mover/synchronous generator combination and conditioning and delivering it in a form convenient for the end-user is very attractive. The form of power distribution for propulsion and ships services is a key element of the overall architecture for any marine electrical power system and it is being explored at present as to whether this should be ac-ac, ac-dc, dc-ac or dc-dc. Much of the evolving thinking, so far as the marine industry is concerned, is chronicled in a series of papers, spreading over a decade now, on the Electric Warship [4,5,6,7,8,9].

However, as with all radical ideas and inventions, new problems will invariably manifest themselves. For example, it may not be clear which of a number of developmental routes to pursue, and an unfortunate choice may well lead to lack of competitiveness or even financial ruin. New equipments may render unforeseen environmental and operational problems, e.g. very large amounts of harmonic power in the system arising owing to the basic switching-mode of operation of the static systems employed.

In addition, many conventional methods of analysis and design are not readily applicable to these proposed systems. Dc systems are in use in, for example, satellite powering arrangements, and they are serious contenders for ship-borne equipments [5]. It is expected also that fuel-cells will eventually become major sources of electrical power generation, thereby making dc-dc systems common.

One of the problems with dc systems is that of stability. They have a propensity to exhibit negative-impedance instability, particularly when seeking to supply constant power loads [10]. There has been some significant work over the last 15 years, or so, in this area by, for example, Middlebrook [10], and Sudhoff et al. [11] that is based, essentially, on frequency-domain techniques.

This present paper forms the first in a series and presents a method for examining stability using root locus methods. Other parts to follow include consideration of frequency-domain techniques, effects of parameter variability, consideration of improving the transient behaviour and non-linear aspects. The authors believe that, rather than produce a general analysis, the application of these techniques to an easily understood problem illustrated in [11] is more enlightening; this will be used as the example throughout the sequence. In this present paper it supports a simple explanation of the design procedure for which the entire computer work was performed with no more than a few lines of code using the MATLAB Version 5.3 package.
2. Analysis

(a) The negative impedance

It is easily appreciated why negative-impedance instability, in a dc system, might be of concern by considering a system delivering constant power to a load. In this case we have, using the usual notation,

\[ P = IV = \text{constant}, \quad (1) \]

Figure 1: Constant Power Operating Characteristic

Geometrically this can be represented by the branch in the first quadrant of a rectangular hyperbola in the V-I plane as in Figure 1. Taking differentials of equation 1, we have.

\[ \Delta P = V \Delta I + I \Delta V = 0, \]

or,

\[ \frac{\Delta V}{\Delta I} = -\frac{V}{I}, \quad \text{say.} \quad (2) \]

That is, the incremental resistance is negative and, further, varies in magnitude with the point of operation of the system.

(b) Basic circuit arrangement

The basic circuit arrangement considered is that shown in Fig. 2, where, in effect, the voltages and currents shown are the incremental values. It is firstly necessary to establish an expression showing how the value of \( V \), the voltage existing across the terminals where the source and load meet, behaves. This is easily done, for example, using Millman's circuit theorem [14] whereby there results,
\[ V = \frac{(V_s, Y_s + V_I, Y_I)}{(Y_I + Y_s)} \]  

(3)

Figure 2: The general circuit arrangement

where,

\[ Y_s = \frac{D_s}{N_s}, \quad Z_s = \frac{N_s}{D_s} \]

\[ Y_I = \frac{D_I}{N_I}, \quad Z_I = \frac{N_I}{D_I} \]

The \( N_i \)'s and \( D_i \)'s are polynomials in \( s \), the Laplace variable, i.e., the \( Z \) and \( Y \) terms are operational impedances and admittances respectively. Substitution of these in equation 3, after some minor algebraic manipulation, leads to

\[ V = \frac{N_s, D_s, V_s + N_I, D_I, V_I}{N_s, D_s, (1 + Z_s, Y_s)} \]

Since neither \( N_i \) nor \( D_i \) have any roots in the right-half plane from passive component consideration, stability is determined by the roots of,

\[ 1 + Z_s, Y_s = 0. \]  

(4)

For a stable system all these roots must be in the left-hand half of the complex plane. Determining the roots of such an equation, as one of the parameters is varied, is known as determining the root-locus [12]. It is convenient to do this graphically (nowadays utilising computer graphics). This technique has been exploited in the mainstream of control engineering work for decades. It is, of course, known as the root-locus technique [13]. However, applications are not restricted to control activities.
The root-locus technique

It is well known that a unity-feedback control system, with a forward transfer-function,

\[ G(s) = \frac{kN(s)}{D(s)}, \]

is stable provided all the roots of the equation,

\[ 1 + G(s) = 1 + k \frac{N(s)}{D(s)} = 0 \]

have negative real parts - where, \( k \) is a gain-term that is varied, (usually from \( k = 0 \) to \( \infty \)), to establish the resulting variation of the roots. This root-locus is usually displayed graphically in the complex plane. If, for example, some particular range of \( k \) places the locus in the right-half of this plane, then the system will be unstable for that range of \( k \).

Further, since the roots obtained by this technique are actually the roots of the characteristic equation, governing the dynamical behaviour of the system, then their position in the complex plane gives immediate insight into the type of behaviour the system will exhibit, e.g. sluggish, oscillatory, etc.

The process of constructing graphically the root-locus is discussed in almost every text on classical control engineering; the root-locus being a major weapon in any control engineer's armoury [12]. In practice computer-graphic methods are now widely available to perform the root-locus construction, and the results are displayed on a VDU. There is nothing that is unique to the control-engineering field about the root-locus technique. It is merely the fact that its provenance is in that field; having been introduced there by Evans in 1948 [13]. In essence it is a convenient graphical technique for determining the roots of any polynomial equation that can be put in the form,

\[ 1 + k \frac{N(s)}{D(s)} = 0, \]

and \( k \) is varied. Indeed, the application to be demonstrated below would hardly be thought of as an essentially control-engineering problem.

The Circuit

The circuit discussed by Sudhoff et al. is shown in Fig. 3 where the voltage across the load is easily established to be governed by the equation,

\[ V = \frac{RV_s}{CRLs^2 + (CRr - L)s + (R - r)} \tag{5} \]

Note that, \( R, C, L \) & \( r \) are all positive quantities. Thus the stability can be examined by finding the position of the roots of the denominator of equation 5 in the complex plane. However, before this is done it is worthwhile performing a Routh-Hurwitz analysis [12] to
determine the limits of stability as specified by relationships between the parameter values. Consider the Routh-Hurwitz table for this denominator.

\[
s^2 \quad CRL \quad (R-r) \\
\]
\[
s \quad CRr-L \quad 0 \\
\]
\[
s^0 \quad (R-r) \\
\]

According to the Routh-Hurwitz criterion the condition, for stability is that there should be no changes in sign of the items in the first column of the table. Since CRL must be positive then (CRr-L) and (R-r) must also be positive for stability.

\[
Rr \quad L, \ r \\
\]
\[
V_s \quad V \quad C \quad -R \\
\]

According to the Routh-Hurwitz criterion the condition, for stability is that there should be no changes in sign of the items in the first column of the table. Since CRL must be positive then (CRr-L) and (R-r) must also be positive for stability.

Figure 3: The Circuit

That is:

\[
C > \frac{L}{Rr} \text{ and } R > r. \\
\]

Critical conditions occur when these inequalities change to equalities. This result could be obtained from quadratic equation theory (see appendix 2); however, for higher-order systems the Routh-Hurwitz criterion can be a very effective and useful technique and hence the demonstration here. The above two inequalities are, of course, exactly as those specified by Sudhoff, [11]. Unfortunately the Routh-Hurwitz criterion indicates only if a system is stable or not, it does not lend itself easily to obtaining further insight into system behaviour. And this is exactly where the root-locus demonstrates not only under what conditions the system is stable, but gives information on the margin of stability and of the characteristic response to be expected from the system. Both of these, of course, depend on the root locations in the complex plane. Essentially, the root locus is a powerful analytical and design technique, but it does require a knowledge of the transfer function under examination.
From the circuit arrangement in Fig. 3, the component values chosen were $r = 300\, \Omega$, $L = 10\, mH$ and $R = 24.3\, \Omega$, so the system is delivering, under constant power conditions, 3.7 kW at 300V, fig. 1. The first problem is to determine suitable values of the capacitor, $C$, to produce acceptable behaviour. We can see that, from the above inequalities the $R>r$ condition is satisfied, and that the critical value of $C$ is given by,

$$C = \frac{L}{rR} = 1.37\, mF$$

Sudhoff’s paper [11] using a development of the Nyquist criterion, concludes that, $C = 40\, mF$, is a stable situation, and, $C = 0.5\, mF$, represents an unstable situation. This is exactly what would be expected from the above inequalities.

Obviously, for stability of an acceptable degree, $C$, must be greater than $1.37\, mF$. The question remains as to what values of, $C$, give desired or, at least, acceptable behaviour. It should, of course, be remembered that large capacity capacitors such as the $0.4F$ capacitor considered here are physically very large and expensive to manufacture.

It would be convenient if there was a graphical way of displaying how the roots of the governing equation vary as $C$ is varied. This is precisely what the root-locus plot can achieve. These roots being, of course, the roots of the characteristic equation for the system.

Now the denominator of equation 5, when equated to zero, is the Laplace transformation of the characteristic equation, viz

$$RCLs^2 + (CRr - L)s + (R - r) = 0 \quad (6)$$

Obviously this equation is not presented in a form suitable for direct application of the root-locus technique, but simple manipulation renders it into such a form; i.e

$$C[RLs^2 + Rrs] - sL - r + R = 0$$

or

$$1 + \frac{(-Ls + R - r)}{CRs(Ls + r)} = 0, \quad (7)$$

which has the form used in control engineering, shown previously, as

$$1 + k \frac{N(s)}{D(s)} = 0,$$

with the $1/C$ term playing the role of the $k$ term. This is also the form required in exploiting MATLAB. At this stage the root-locus commands for the computer programme MATLAB (version 5.3) were utilised in displaying this root-loci on a VDU. The plot resulting can be interrogated for specific value of $k$ (i.e. $1/C$) and the corresponding roots. This whole process can be done with less than ten lines of computer code.
So entering the specific values of the coefficients of the numerator and denominator polynomials of equation (7), for the problem in hand, into the r locus (n,p) command of MATLAB, leads to Fig. 4a. (Note that there is a commentary on all the figures in appendix 1. Because of the values of the coefficients, the plot is interesting but not particularly useful for the purpose required, ie the range of C required to ensure stability. The values of the two poles of the equation 7 are actually situated on the real axis at -30 and 0 but they look to be coincident in Fig. 4a. It can be seen, however, that only a small portion of the root-locus that to the left of the imaginary axis, indicates a stable region. By using the zoom facility provided by MATLAB the area of interest shown in Fig. 4b can be examined in detail. Since roots of the equations, if complex, occur in complex pairs the part of the root-locus below the real axis is shown notionally.

The rlocfind command provided by MATLAB prints out the value of any roots specified by the cursor on the computer screen, as well as the value of k (or 1/C in our case). Fig. 4b shows seven specific points on the graph the figure legend specifies the values of C found for these particular points and the corresponding damping factor [12].

What is seen immediately from this plot is that increasing the capacitor value increases the damping effect and decreases the frequency of the oscillation. As will become apparent, roots along the real axes are achievable only with very large, and unrealistic, values of capacitance.
Although one can glean from the position of the roots the sort of response expected from different values of $C$, MATLAB has a facility for displaying the response to a unit-step (used in this study) and unit-impulse disturbing a system. In the present application we wish to disturb the system in some way to observe what happens. As an illustration this contribution shows what happens when a step-disturbance is introduced in the supply voltage. Because of the MATLAB facilities this can be simulated with very little effort on the designer's part.

Fig. 5a predicts this response when $C = 0.44$ F. The response for this value is critically-damped, i.e. it has two equal real roots. However, this capacitor value is large. Although the root-locus has branches along the real axis, the roots here can only be obtained with even larger capacitor values, and so are of very limited interest in practice.

Fig 5b predicts the response when employing a capacitor of 36.5 mF value. The response is now oscillatory, over-shooting by about 40% but the transient has virtually died out in 0.35 s. Using a capacitor of value 8.1 mF, Fig. 5c, it can be seen that this overshoot is approximately 72% and the transient is virtually over in 0.5 s. It can also be seen that the frequency of the oscillation has increased markedly from that of the 2.0 mF case.

Fig. 5d shows the result obtained using a capacitor of value $C=4.1$ mF. The overshoot is about 82%, but again the transient is effectively over in 0.5 s.
Fig. 5e shows the response with a capacitor value of \( C = 2.0 \) mF. It is seen to have a much more oscillatory character. The overshoot is some 95\%, the frequency of oscillation has increased markedly from that of the previous case and there is still perceptible oscillation after 1s.

The value of the capacitor at which sustained oscillation is predicted is \( C = 1.37 \) mF. Fig. 5f indicates the response for a value of capacitor of 1.4mF (slightly above this critical value) but continuous oscillation is seen to be very close indeed.

This series of graphs showing the behavioural changes with decreasing capacitance is what one could predict roughly from a preliminary viewing of the root-locus curve. The root-locus not only indicates critical values of components to ensure stability, but indicates, at a glance, the main characteristic features of the responses to be expected.

Two features are particularly interesting. The first is that a change of capacitance from 0.44F to 20mF changes the real-part of the characteristic roots from 15 rad/s to 14 rad/s. However, the imaginary parts change markedly.

The second feature is that in the stable region where the locus is moving rapidly towards unstable behaviour, for example, from 2.0mF to 1.4mF, the decay decreases very markedly, but the oscillatory nature of the responses do not change much in frequency.

(e) **Behaviour with different value of load resistance, \( R \)**

Depending upon operating conditions, the magnitude of the incremental resistance, \( R \), will vary even if the power delivered is constant. To assess how the value of \( R \) affects the behaviour of the system, the root-locus can once more be used. This can be done by keeping the value of the capacitor constant, at some specific value, but this time the \( R \) can be used as the parameter of variation.

The characteristic equation can be manipulated to the required form by collecting all terms in \( R \) together, viz rearrange equation 6 into the form

\[
R [CLs^2 + CLr + 1] - sL - r = 0
\]
This can be re-written,

\[ 1 + \frac{1}{R} \cdot \frac{(-Ls - r)}{CLs^2 + Crs + 1} = 0 \]

Thus the term \(1/R\) is the "k-term" in the standard control engineering expression governing the characteristic equation in the form suitable for root-locus investigation.

Figure 7a: Time response. \(R = 48.4\ \Omega\)

Figure 7b: Time response. \(R = 24.3\ \Omega\)

Figure 7c Time response. \(R = 15\ \Omega\)

Figure 7d: Time response. \(R = 12\ \Omega\)

Figure 7e Time response. \(R = 9.6\ \Omega\)

Figure 7f Time response. \(R = 8.2\ \Omega\)
Using a value of \( C = 4.1 \) mF, the root-locus diagram shown in Fig 6a is obtained. Once again it is only a small portion of the plot that is of real interest, i.e. the portion to the left of the imaginary axis. Using the zoom command of MATLAB produces Fig. 6b. Since any complex roots occur in conjugate pairs it is sufficient to plot the branches, indicating these, for the top half-plane above the real axis. This area is shown calibrated by indicating six different values of \( R \).

An interesting feature is that the period of oscillation is very insensitive to the value of \( R \) in the region of interest, but the damping is extremely sensitive. Fig. 7 show the system response for various values of \( R \) when a step-disturbance is made to the supply voltage, \( V \).

It can be seen as the value of \( R \) decreases the system moves towards instability, indeed, with \( R = 8.2 \Omega \) the limit of stability has practically been reached. The results support the notion that when delivering constant power it helps, so far as stability is concerned, to run at as high a voltage as is practically convenient, since this increases the value of \( R \).

3. Conclusion

It has been demonstrated that the root-locus technique can be a useful tool in dealing with design and stability assessment of dc distribution systems. This has been verified using a particular example, but is generally applicable to any such system whose behaviour may be assumed linear (at least around some operating point), and whose dynamical equation is known. The availability of computer programs, such as MATLAB, enables the process to be carried out without any calculation being done by the designer. Information concerning suitable parameter values and the corresponding characteristic-root values are immediately available by interrogating the displayed root-locus. The response of the system to step changes in, say, the supply voltage is also immediately available using the MATLAB software.

4. Acknowledgement

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5. References


Appendix 1

Commentary on Figures

1. Characteristic curve for the constant power load

Values of variables are those for the system considered in Section 3.

2. Figure 2

This is a schematic of a general system where the terms used have the following meanings:

- \( V_s \) - supply voltage
- \( V_l \) - voltage generated by load (assumed to be zero in this paper)
- \( V \) - voltage at the junction of the supply and load sub-systems
- \( Z_s \) - source impedance, \( Y_s \) - source admittance
- \( Z_l \) - load impedance, \( Y_l \) - load admittance

3. Figure 3

This is the circuit diagram of the illustrative example and matches that used by Sudhoff in [11]. Note that there is no generated load voltage. Effectively the variables and component values represent their perturbational values, and the load impedance becomes \(-R\).

4. Figure 4

Both Figure 4a and Figure 4b show the root-locus plot illustrating the effect of changing capacitor values with \( R = 24.3 \ \Omega \). Figure 4a gives the complete root locus plot. There are two poles (at 0 rad/s and \(-30\) rad/s) and one zero (at 2400 rad/s). Only the small region around the poles indicates a stable system Figure 4b gives the detail of the stable region. For a realisable physical systems complex roots occur in conjugate pairs, hence only the upper half-plane plot in this region is shown. The plot is calibrated showing seven specific values of \( C \) the capacitor value as given in the Table below.

<table>
<thead>
<tr>
<th>Root Position No</th>
<th>Capacitor Value</th>
<th>Damping Factor</th>
<th>Theoretical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.44F</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>36.5mF</td>
<td>0.280</td>
<td>0.278</td>
</tr>
<tr>
<td>3</td>
<td>8.1mF</td>
<td>0.110</td>
<td>0.112</td>
</tr>
<tr>
<td>4</td>
<td>4.1mF</td>
<td>0.064</td>
<td>0.0643</td>
</tr>
<tr>
<td>5</td>
<td>2.0mF</td>
<td>0.021</td>
<td>0.0212</td>
</tr>
<tr>
<td>6</td>
<td>1.4mF</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1.0mF</td>
<td>unstable</td>
<td></td>
</tr>
</tbody>
</table>
5. **Figure 5**

Figures 5a to 5f show representative step responses for the various values of C as used in Figure 4b except that the unstable response for position 7 (C = 1.0 mF) is not given. The step change occurs as Vs in Figures 2 and 3.

6. **Figure 6**

Both Figure 6a and Figure 6b show the root-locus plot illustrating the effect of changing load resistance values with C = 4.1 mF. Figure 6a gives the complete root locus plot. There are two poles (at −15 rad/s ± j156 rad/s) and one zero (at −30 rad/s). Only the locus positioned in the left-half plane indicates stable behaviour Figure 6b gives the detail of the stable region. The plot is calibrated showing six specific values of the perturbational load resistor as listed in the table below.

<table>
<thead>
<tr>
<th>Root Position</th>
<th>Resistor Value (negative)</th>
<th>Damping Factor</th>
<th>Theoretical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48.4 Ω</td>
<td>0.080</td>
<td>0.0822</td>
</tr>
<tr>
<td>2</td>
<td>24.3 Ω</td>
<td>0.064</td>
<td>0.0643</td>
</tr>
<tr>
<td>3</td>
<td>15.0 Ω</td>
<td>0.044</td>
<td>0.0444</td>
</tr>
<tr>
<td>4</td>
<td>12.0 Ω</td>
<td>0.032</td>
<td>0.0314</td>
</tr>
<tr>
<td>5</td>
<td>9.8 Ω</td>
<td>0.016</td>
<td>0.0166</td>
</tr>
<tr>
<td>6</td>
<td>8.2 Ω</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

7. **Figure 7**

Figures 7a to 7f show representative step responses for the various values of R as used in Figure 6b. As before the step change occurs as Vs in Figures 2 and 3.
Appendix 2

Alternative derivation on condition for stability

Equation 3 may be written,

$$CRLs^2 + (CRr - L)s + (R - r) = 0$$

This system is stable provided the roots of equation A1 have negative real parts.

Since all component values are positive therefore CRL is positive. Provided the remaining coefficients are positive then by the Descartes' rule of signs the system must be stable. (Unfortunately this is only a necessary condition).

That is

$$CRr - L > 0, \text{ i.e. } C > \frac{L}{Rr},$$

and $$R - r > 0, \text{ i.e. } R > r.$$

Alternatively since equation A1 is a quadratic, its roots may be written

$$s_{12} = -\frac{(CRr - L) \pm \sqrt{(CRr - L)^2 - 4CRL(R - r)}}{2CRL}$$

The first condition to be satisfied, if the roots are to have negative real-parts is

$$CRr - L > 0,$$

i.e. $$C > \frac{L}{Rr}$$

Consider now the term under square-root sign. This will always be less than $$(CRr - L),$$ provided $$4CRL(R - r)$$ remains positive. This implies that $$R > r$$ for stability. If this inequality is not satisfied, then the positive value of the square-root will be greater than $$(CRr - L),$$ thereby implying one positive-valued root. Thus the inequalities

$$C > \frac{L}{rR} \text{ and } R > r$$

are actually necessary and sufficient for stability, in this particular case.